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10MAT21

Second Semester B.E. Degree Examination, Dec.2016/Jan.2017
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, choosing at least two from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) The general solution of $p^2 - 7p + 12 = 0$ is,
A) $(y+3x-c)(y+4x-c)=0$ B) $(y-3x-c)(y-4x-c)=0$
C) $(y-4x)(y+3x)=0$ D) None of these

ii) If a differential equation is solvable for y then it is of the form,
A) $x = f(y,p)$ B) $y = f(x,p)$ C) $y = f(x^2, py)$ D) $x = f(y^2, p)$

iii) The singular solution of the equation $P = \log(px - y)$ is,
A) $y = x(\log x - 1)$ B) $y = 1 - \log x$ C) $y = \log x - 2x$ D) $y = 1 - \log\left(\frac{1}{x}\right)$.

iv) Clairaut's equation of $P = \sin(y - xp)$ is,
A) $y = \frac{P}{x} + \sin^{-1} p$ B) $y = px + \sin p$ C) $y = px + \sin^{-1} p$ D) $y = x + \sin^{-1} p$

b. Solve: $p(p+y) = x(x+y)$. **(05 Marks)**

c. Solve : $y = 2px + y^2 p^3$. **(05 Marks)**

d. Obtain the general solution and singular solution of the equation,
$\sin px \cos y = \cos px \sin y + p$ **(06 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) Roots of $y'' - 6y' + 13y = 0$ are,
A) $2 \pm 3i$ B) $2 \pm i$ C) $3 \pm i$ D) $3 \pm 2i$

ii) The value of $\frac{1}{D}(f(x))$ is,
A) $f'(x)$ B) $\frac{1}{f'(x)}$ C) $\int \frac{1}{f(x)} dx$ D) $\int f(x) dx$

iii) The complementary function for the differential equation, $y'' + 2y' + y = \cosh x$ is,
A) $(c_1 + c_2 x)e^{-x}$ B) $c_1 e^{-x} + c_2 e^{x}$ C) $(c_1 + c_2 x)e^{x}$ D) $(c_1 + c_2)e^{-x}$

iv) The particular integral of $(D^2 - 2D + 4)y = e^x \cos x$ is,
A) $e^x \sin x$ B) $\frac{e^x \cos x}{2}$ C) $\frac{e^x \sin x}{2}$ D) None of these

b. Solve $(4D^4 - 4D^3 - 23D^2 + 12D + 36)y = 0$ where $D = \frac{d}{dx}$. **(05 Marks)**

c. Solve : $(D^2 - 2D + 5)y = e^{2x} \sin x$. **(05 Marks)**

d. Solve : $\frac{dx}{dt} - 2y = \cos 2t$, $\frac{dy}{dt} + 2x = \sin 2t$ given that $x = 1$, $y = 0$ at $t = 0$. **(06 Marks)**

3 a. Choose the correct answers for the following : **(04 Marks)**

i) By the method of variation of parameters, the value of W is called,
A) the Demorgan's function B) Euler's function
C) Wronskian of the function D) None of these

ii) In $x^2 y'' - xy' + y = x^2 \log x$ if $x = e^t$ then we get for $x^2 y''$ as,
A) $(D-1)y$ B) $D(D-1)y$ C) $D(D+1)y$ D) $D(D+2)y$

iii) To transform $(ax+b)^2 y'' + K_1(ax+b)y' + K_2 y = \phi(x)$ into Legendre's linear equation we put $(ax+b) =$ __________

A) e^{-t} B) $1+e^{t}$ C) $\frac{1}{e^{-t}}$ D) $1-e^{t}$

iv) Frobenius series method of second order linear differential equation is of the form,

A) $x^m \sum_{r=0}^{\infty} a_r x^r$ B) $\sum_{r=0}^{\infty} a_r x^r$ C) $\sum_{r=0}^{\infty} a_r x^{m-r}$ D) None of these

b. Solve $(D^2+1)y = \operatorname{cosec} x \cot x$ by the method of variation of parameters. **(05 Marks)**

c. Solve: $x^2 y'' + xy' + 9y = 3x^2 + \sin(3\log x)$. **(05 Marks)**

d. Obtain the series solution of the equation, $\frac{d^2y}{dx^2} + xy = 0$. **(06 Marks)**

4 a. Choose the correct answers for the following : **(04 Marks)**

i) Form the partial differential equation by eliminating a and b from the relation $z = (x+a)(y+b)$ is,

A) $z = pq$ B) $z = p+q$ C) $z = 1+p$ D) $z = 1+q$

ii) The solution of $u_{xx} = x + y$ is $u =$ ________ .

A) $\frac{x^3}{6} + \frac{x^2 y}{2} + xf(y) + g(y)$ B) $\frac{x^3}{4} + \frac{x^2 y^2}{2} + yf(x) + g(y)$

C) $\frac{xy}{3} + \frac{xy}{4} + yf(y) + g(x)$ D) None of these

iii) The auxiliary equations of Lagrange's linear equation, $Pp + Qq = R$ are,

A) $\frac{dx}{P} = \frac{dy}{q} = \frac{dz}{R}$ B) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ C) $\frac{dx}{P} = \frac{-dy}{Q} = \frac{dz}{R}$ D) $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$

iv) In the method of separation of variables to solve $u_x = 2u_t + u$, the trial solution is $u =$ _____ .

A) $X(x)Y(y)$ B) $X+Y$ C) $Z = X^2 + Y^2$ D) $X(x)T(t)$

b. Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$ given that when $x = 0$, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$. **(05 Marks)**

c. Solve : $x(y^2+z)p - y(x^2+z)q = z(x^2 - y^2)$. **(05 Marks)**

d. Solve by the method of separation of variables $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x,0) = 6e^{-3x}$ **(06 Marks)**

PART – B

5 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of $\int_0^1 \int_0^6 xy\,dx\,dy$ is ________

A) 6 B) 7 C) 8 D) 9

ii) The integral $\int_0^1 \int_0^{\sqrt{1-x^2}} (x+y)\,dy\,dx$ after changing the order of integration is ______,

A) $\int_0^2 \int_0^{\sqrt{1-y^2}} (x+y)\,dx\,dy$ B) $\int_0^1 \int_0^{\sqrt{1-y^2}} (x+y)\,dx\,dy$ C) $\int_0^1 \int_0^{\sqrt{1+y^2}} (x+y)\,dx\,dy$ D) None of these

iii) The value of $\int_0^{\infty} e^{-x^2}\,dx$ is ________

A) $\pi\sqrt{2}$ B) $2\sqrt{\pi}$ C) $\sqrt{2\pi}$ D) $\sqrt{\pi}/2$

iv) In terms of Beta function $\int_0^{\frac{\pi}{2}} \sin^7\theta\sqrt{\cos\theta}\,d\theta =$ ________

A) $\beta\left(4,\frac{3}{4}\right)$ B) $\frac{1}{2}\beta\left(4,\frac{3}{4}\right)$ C) $\beta\left(2,\frac{3}{2}\right)$ D) $\frac{1}{2}\beta\left(2,\frac{3}{2}\right)$

(05 Marks)

b. Evaluate $\int_0^{\infty}\int_0^{x} xe^{\frac{-x^2}{y}}\,dydx$ by changing the order of integration. (05 Marks)

c. Evaluate $\int_0^{a}\int_0^{x}\int_0^{x+y} e^{x+y+z}\,dzdydx$. (05 Marks)

d. Express the integral $\int_0^1 x^m(1-x^n)^p\,dx$ in terms of beta functions and hence evaluate $\int_0^1 x^5(1-x^3)^{10}\,dx$. (06 Marks)

6 a. Choose the correct answers for the following : (04 Marks)

i) If $\int_C \vec{F}\cdot d\vec{r} = 0$ then $\vec{F}$ is called,

A) Rotational B) Solenoidal C) Irrotational D) Dependent

ii) If $f = (5xy-6x^2)i + (2y-4x)j$ then $\int_C f.dr$ where C is the curve $y = x^3$ from the points (1, 1) to (2, 8) is,

A) 35 B) -35 C) 3x+4y D) None of these

iii) In Green's theorem in the plane $\iint_A\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)dxdy$ is ________

A) $\iint_C (Mdx - Ndy)$ B) $\int_C (Mdx)\times(Ndy)$ C) $\int_C (Ndx - Mdy)$ D) $\int_C (Mdx + Ndy)$

iv) If C be a simple closed curve in space and S be the open surface, f be the vector field then $\int_C f\cdot dv =$ ________

A) $\int_S (\text{curl} f)\cdot nds$ B) $\int_C (\nabla\times f)\cdot ds$ C) $\int_S (\nabla^2 f)\cdot nds$ D) $\int_C (\nabla\cdot f)\cdot nds$

b. If $f = (3x^2+6y)i - 14yzj + 20xz^2k$ evaluate $\int_C f\cdot dr$ from (0, 0, 0) to (1, 1, 1) along the curve C given by $x = t, y = t^2, z = t^3$. (05 Marks)

c. Verify Green's theorem for $\int_C (xy+y^2)dx + x^2dy$ where C is the closed curve made up of the line $y = x$ and the parabola $y = x^2$. (05 Marks)

d. If $\vec{F} = 2xyi + yz^2j + xzk$ and S is the rectangular parallelopiped bounded by x = 0, y = 0, z = 0, x = 2, y = 1, z = 3 evaluate $\iint_S \vec{F}\cdot\hat{n}ds$. (06 Marks)

7 a. Choose the correct answers for the following : (04 Marks)

i) $L\{\sinh at\} =$ ______

A) $\frac{a}{s^2+a^2}$ B) $\frac{s}{s^2-a^2}$ C) $\frac{a}{s^2-a^2}$ D) $\frac{s}{s^2+a^2}$

ii) $L\{t^2 e^{-at}\} =$ ______

A) $\frac{1}{(s+a)^3}$ B) $\frac{2}{(s+a)^2}$ C) $\frac{3}{(s+a)^3}$ D) $\frac{2}{(s+a)^3}$

iii) Transform of unit function $L\{u(t-a)\} =$ ______

A) $\frac{e^{as}}{s}$ B) $\frac{e^{-as}}{s^2}$ C) $\frac{e^{-as}}{s}$ D) $\frac{e^{as}}{s^2}$

iv) $L[\delta(t-a)]$ is equal to,

A) 0 B) -1 C) e^{-as} D) e^{as}

b. Find $L\left[\frac{1-\cos 3t}{t}\right]$. (05 Marks)

c. Find $L\{f(t)\}$ where $f(t) = \begin{cases} 3t, & 0<t<2 \\ 6, & 2<t<4 \end{cases}$ where the period is 4. (05 Marks)

d. Express $f(t)$ in terms of unit step function and hence find $L\{f(t)\}$ given that $f(t) = \begin{cases} t^2, & 0<t\le 2 \\ 4t, & t>2 \end{cases}$. (06 Marks)

8 a. Choose the correct answers for the following : (04 Marks)

i) $L^{-1}\left\{\frac{s}{s^2-16}\right\} =$ ______

A) $\cosh 4t$ B) $\sinh 4t$ C) $\frac{1}{4}\cos 4t$ D) None of these

ii) $L^{-1}\left\{\frac{s+1}{s^2+6s+9}\right\} =$ ______

A) $e^{3t}(1+2t)$ B) $e^{-3t}(1-2t)$ C) $e^{-3t}(1+2t)$ D) $e^{-3t}(1+t)$

iii) $L^{-1}\left\{\cot^{-1}\left(\frac{s}{a}\right)\right\} =$ ______

A) $\frac{\sin t}{t}$ B) $\frac{\sinh at}{t}$ C) $\frac{\sin at}{t}$ D) $\frac{\sinh t}{t}$

iv) $L^{-1}[\bar{f}(s)\cdot\bar{g}(s)] =$ ______

A) $f(t)\cdot g(t)$ B) $\int_0^t f(u)g(t-u)du$ C) $\int_0^t f(u)g(u)du$ D) None of these

b. Find $L^{-1}\left[\frac{4s+5}{(s+1)^2(s+2)}\right]$. (05 Marks)

c. Find $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$ using convolution theorem. (05 Marks)

d. Solve $y''(t)+4y'(t)+4y(t)=e^{-t}$ with $y(0)=0$ and $y'(0)=0$ using Laplace transform method. (06 Marks)

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